SYNOPSIS FOR THESIS

"Dynamics of Disease Transmission through Mathematical Models"

Submitted to

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Nanded- 431606.

स्वामी रामानंद तीर्थ मराठवाडा विद्यापीठ, नांदेड.

For the award of the Degree of

Doctor of Philosophy (Ph.D.)
In Mathematics

Under the

Faculty of Science and Technology

Submitted by

Mr. Urekar Balaji Gangadhar
M.Sc. SET (Mathematics)

Under the guidance of

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Head, Dept. of Mathematics
Digambarrao Bindu ACS College, Bhokar,
Dist. Nanded (Maharashtra).

To,

The Deputy Registrar (P.G. Section),

S. R. T. M. University,

Nanded.

Subject: Permission to submit the Synopsis for Doctor of Philosophy (Ph. D.**) in Mathematics.**

Respected Sir,

I am recognized guide in Mathematics of our University S. R. T. M. U. Nanded. My candidate **Mr. Urekar Balaji Gangadhar** is submitting registration form for Ph. D. under my guidance. So kindly permit to submit the Synopsis.

All required and needful documents will be fulfilled by my candidate.

Thanking you.

Yours Faithfully

Dr. S. B. Chavhan

Head, Dept. of Mathematics
Digambarrao Bindu ACS College, Bhokar,
Dist- Nanded (Maharashtra).

To,

The Director,

School of Mathematical Sciences,

S. R. T. M. University,

Nanded.

Subject: Permission to Submit Registration Form of Mr. Urekar Balaji Gangadhar.

Respected Sir,

With reference to subject cited above, sir, I am recognized guide in Mathematics of our research center. My candidate Mr. Urekar Balaji Gangadhar is submitting registration form for Ph. D. under my guidance through our research center.

So kindly permit to submit the form.

All required needful documents will be fulfill by my candidate.

Thanking you.

Yours Faithfully

Dr. S. B. Chavhan

Head, Dept. of Mathematics
Digambarrao Bindu ACS College, Bhokar
Dist- Nanded (Maharashtra).

“Dynamics of Disease Transmission through Mathematical Models”

1. Introduction:-

The spread of infectious diseases can be unpredictable. With the emergence of antibiotic resistance and worrying new viruses, and with ambitious plans for global eradication and the elimination of infectious diseases such as polio, malaria, tuberculosis etc., the stakes have never been higher. Anticipation and measurement of the multiple factors involved in infectious disease can be greatly assisted by mathematical methods. Mathematical model is a description of a system using mathematical tools and language. The process of developing mathematical models is called mathematical modelling. We will be concerned with modelling infectious diseases and their spread in populations, but in principle, mathematical modelling can be applied to any system, biological or otherwise. In particular, Mathematical modelling techniques can help to compensate for imperfect knowledge, gathered from large populations and under difficult prevailing circumstances.

The modelling process requires translation of a biological scenario into a mathematical problem. The modelling process typically begins with a clear description of the processes based on the scientist’s understanding of the system. Epidemic dynamics is an important method of studying the spread of infectious disease qualitatively and quantitatively. It is based on the specific property of population growth, the spread rules of infectious diseases, and the related social factors, etc.

Mathematical models in epidemiology generally come in two forms: stochastic or deterministic. Simulations of stochastic models often require more computational work, but can reflect the environmental and demographic randomness that is prevalent in epidemiological systems. Deterministic models (such as the SIR model , which often involve systems of differential equations (DEs), generally not only require less computational work to obtain a solution (or at least to approximate a

solution using numerical methods), but one can also use many tools from DE analysis (such as linear stability analysis and Floquet theory to explore their behaviour.

2. Review of Research and development in the subject:-

While mathematical modelling of infectious diseases can be traced back to 1760 when Bernoulli used mathematical models for smallpox [3], the research on infectious diseases, using deterministic mathematical models, actually began in the 20th century. Hamer formulated a discrete-time model for the spread of measles in 1906. A physician, Dr. Ross, used a differential equation model to describe the transmissions of malaria between human beings and mosquitoes in 1911, and determined that there exists a threshold of the size of mosquitoes below which the spread of malaria can be controlled. It was because of his outstanding contributions in the research of the transmission dynamics of malaria Dr. Ross was awarded his second Nobel Prize in medicine.

Kermack and McKendrick [21, 22] formulated a well-known and well-recognized SIR (susceptible-infective–recovered) compartmental model, in 1926, to study the outbreak of Black Death in London during the period of 1665–1666, and the outbreak of plague in Mumbai in 1906 and corresponding model equations are given in the system as follows,

$$\frac{dS}{dt} = -\beta SI,$$
$$\frac{dI}{dt} = \beta SI - \gamma I,$$
$$\frac{dR}{dt} = \gamma I.$$

Where, at time t, S = Susceptible Compartment, I = Infected Compartment,
R = Removed or Recovered Compartment, γ- Recovery rate coefficient and β- Number of susceptibles with the proportional coefficient (transmission coefficient).

In middle 1800's Louis Pasteur confirmed experimentally the germ theory of disease and he created the first vaccine for rabies. At the same time, Robert became famous for the discovery of the anthrax bacillus (1877), the tuberculosis bacillus (1882)

and the cholera vibrio (1883) and for his development of Koch's postulates. Recently In [23], Blessing. O. Emerenini and Simeon. C. Inyama developed mathematical model for the spread of Hepatitis B virus. The model consist of five governing differential equations is given by

$$\frac{dM(t)}{dt} = cP - \emptyset M - \beta M$$

$$\frac{dS(t)}{dt} = (1-c)P + \emptyset M + \pi R - kS - \beta S$$

$$\frac{dL(t)}{dt} = kS - qL - \mu L - \beta L$$

$$\frac{dI(t)}{dt} = \mu L - \psi \mathrm{I} - \eta \mathrm{I} - \beta \mathrm{I}$$

$$\frac{dR(t)}{dt} = qL - \psi I - \pi R - \beta R$$

$$N(t) = M(t) + S(t) + L(t) + I(t) + R(t)$$

Where ϕ - Rate of expiration of vaccine efficacy

k - Rate at which susceptible individuals become latently infected by HB

μ - The rate at which latently infected individuals become actively infected

ψ - Rate at which actively infected individuals become recovered from HB infection

η - HB- induced mortality,

β - Natural mortality

P - Population of new births

cP - The proportion of new births that have been immunized through vaccination and

N - the total population size.

More developments and progresses have been particularly made during the past 30 years. Massive mathematical models have been formulated and developed to study various infectious diseases, ranging from more theoretic, general ones Waltman [20]; Burnett and White [6]; Hoppensteadt [17]; Frauenthal [14]; Anderson and May[1]; Evans[13]; Busenberg and Cooke[8]; Capasso[9]; Isham and Medley[19]; Daley and Gani[11]; Diekman and Heesterbeek [12] to more specific ones especially for measles, malaria, tuberculosis, sexually transmitted diseases (STD), or AID/HIV [4,5,15,16,18,].

3. Origin of the research problem:-

The first significant epidemic described by historians was the plague of Athens, which struck the city of Athens in 430–426B.C.E. Another disastrous epidemic attacked the Aztec population in the sixteenth century. This smallpox epidemic killed an estimated 35 million people. In the early twentieth century, an influenza pandemic killed an estimated 20million of the world's population. At present, we still have significant outbreaks of epidemics: The Bombay plague 1905–1906, the 2003 severe acute respiratory syndrome (SARS), and the H1N1 swine flu pandemic of 2009. The first statistical study of infectious diseases is attributed to John Graunt (1620–1674). A century later, Daniel Bernoulli used mathematical methods to analyze mortality from smallpox. In 1760, he published what is now considered the first epidemiological model [3].

In the mid nineteenth century, Louis Pasteur made remarkable breakthroughs in the causes and prevention of disease. He reduced mortality from puerperal fever and created the first vaccines for rabies and anthrax. Mathematical modelling of infectious diseases gained importance in the 1980s with the advent of the HIV epidemics. Since then, a very large number of models have been created, analyzed, and employed to study the spread of infectious disease. Today, mathematical epidemiology has a steady presence in the research literature, and mathematical modelling is making significant contributions to Mathematics and public health.

4. Significance of the study:-

Mathematical models are of great importance in the natural sciences, including biology and epidemiology. They help us to gain new understanding about a system, organize and make sense of biological data, obtain the response behaviour of the system, seek optimal performance and intervention strategies, and make predictions about the system.

Mathematical modelling in epidemiology provides understanding of the underlying mechanisms that influence the spread of disease and, in the process, it suggests control strategies. In fact, models often identify behaviours that are unclear in experimental

data – often because data are non-reproducible and the number of data points is limited and subject to errors in measurement. Most mathematical epidemic models usually exhibit "threshold" behaviour which consistent with observations and quantified via epidemiological models, has been consistently used to estimate the effectiveness of vaccination policies and the likelihood that a disease may be eliminated or eradicated.

Hence, even if it is not possible to verify hypotheses accurately, agreement with hypotheses of a qualitative nature is often valuable.

5. Objectivities of the Research:-

1. To propose new Mathematical models for some infectious diseases and investigate their
    i. Steady spatially periodic structures
    ii. Periodic standing wave solutions
    iii. Periodic travelling wave solutions etc.
2. To construct mathematical models reflecting the dynamic properties of infectious diseases.
3. To analyze the dynamical behavior and to do some simulations.
4. We also investigate the qualitative properties such as stability and boundedness of the solutions of the proposed Mathematical models
5. To give Stability of the equilibrium points and to discuss epidemic dynamics for different values of parameters.
6. To find the three threshold quantities:
    i. **The basic reproduction number R_0**, which is the number of secondary cases, produced in a totally susceptible population by a single infective individual during the time span of infection.
    ii. **The contact number σ** which is the average number of adequate contacts of a typical infective during the infectious period

iii. **The replacement number R** which is the average number of secondary infections produced by a typical infective during the entire period of infectiousness.

6. Methodology:-

1. We study of the existing Mathematical models such as SIR, **SISVS**, SIQS, SIQR, etc.
2. We discuss the qualitative properties of solutions of the existing models.
3. We use integral and differential equations tools to frame the models.
4. If necessary, we may use probability distributions while modelling the simple stochastic epidemic models.
5. Both stochastic and deterministic modeling approaches will be utilized.
6. Use many tools from DE analysis such as linear stability analysis and Floquet theory to explore the behavior of the solutions.
7. Use of numerical methods to approximate a solution wherever exact solution is not possible.

7. Plan of Research Work:-

Sr. No	Period	Plan of the work	Targets to be achieved
1	0-6 months	Study of the background required for the research topic.	Good command on the topic.
2	7-12 months	Analyzing the existing models	To derive qualitative properties
3	13-17 months	Framing of new models	Study of stability and asymptotic behaviour of the solutions
4	18-24 months	Summary of the research work	Submit some more papers and Thesis submission.

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